ISI – Bangalore Center – B Math - Physics III –Back Exam
Date: 1 Jan 2020. Duration of Exam: 3 hours
Total marks: 70

Q 1. [Total Marks: 8+7 = 15]

A non conducting spherical shell of radius a and negligible thickness is centered at the origin. It carried a net positive charge $3Q$. A second larger shell of finite thickness is also centered at the origin and has inner radius b and outer radius c. The outer shell is made of conducting material and has no net charge.

1a.) Determine $V(r)$ as a function of r. Plot $V(r)$ vs. r showing clearly how the function behaves as r changes.

1b.) Calculate the total electrostatic energy of the system.

Q2. [Total Marks: 2+5+3 = 10]

2a.) Describe the "method of image" for finding the electrostatic potential in presence of charges and conductors and explain why the method works.

2b.) Find the charge density induced on the surface of an infinite grounded conducting plane in the presence of a point charge q kept at d distance from the conducting plane.

2c.) Calculate the force exerted upon a point charge q held at a distance d in front of a grounded conducting plane.

Q3. [Total Marks: 6+6 = 12]

3a.) A steady current I flows a long cylindrical wire of radius a.
Find the magnetic field both inside and outside the wire, if (i) If the current is uniformly distributed over the surface of the wire and (ii) the current is distributed in such a way that the current density J is proportional to s, the distance from the axis.

3b.) Find the magnetic field at the center of a square loop that carries a steady current I. Let R be the distance from the center to any side.

Q4. [Total Marks: 4+8+4=15]

Suppose a parallel plate circular capacitor of radius *a* is being charged with a steady current *I*. Assume that the charge deposited on the capacitor plates is immediately uniformly distributed over the capacitor plates. Also assume the capacitor plates are close enough so that the electric filed can be taken as perpendicular to the surface.

4a.) Show that the electric field as a function of time is given by $\frac{It}{\pi\varepsilon_o a^2}\hat{z}$.

4b.) Show that the magnetic field field at a point P, see the attached picture for Q4, which is equidistant from the capacitor plates, denoted by coordinates $(s,\phi,z=0)$, is given by

$$\frac{\mu_0 I}{2\pi a^2}s\hat{\phi}$$

4c.) Calculate the Poynting vector $\vec{P}=\frac{\vec{E}x\vec{B}}{\mu_0}$ at the above point and explain its direction in physical terms.

Q 5. [Total Marks: 2+3+4+4+ 5=18]

5.a)Write the complete set of Maxwell's equations in differential form in terms of charge and current densities without any assumption of material response to such fields.

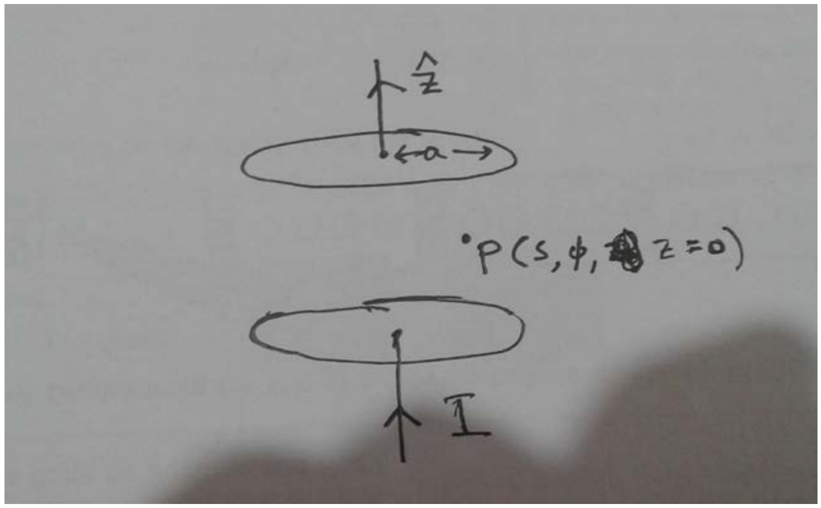
5b.)Derive using the Maxwell's equations, the continuity equation which implies the conservation of charge.

5c.) Show that the Maxwell's equations imply the following

$\vec{B}=\nabla \text{ x } \vec{A}$ and $\vec{E}=-\nabla V-\frac{\partial\vec{A}}{\partial t}$ where V and $\vec{A}$ are scalar and vector potentials.

5d.) How are the above Maxwell's equations modified in the presence of material which show linear response to electric and magnetic materials, and free charges and current?

5e.) Show that in any charge free region of space both electric and magnetic field both obey a wave equation and determine its speed in terms of the parameters present in the Maxwell's equations.



Picture for Question 4